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Measuring Protection

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Measuring Protection

1. The Factor - service indifference curve

Measuring the effects of protection requires a very powerful model, one which can handle any number of goods, factors, and countries and all types of goods and factors. It must also be able to handle foreign investment and the migration of factors other than capital. Partial equilibrium models, like that of the implicit tariff school, or small general equilibrium models, capable of treating only one or two goods, factors, or countries, are clearly inadequate. Until now, however, the great complexity of even simple trade models has prevented the construction of more realistic ones.

Fortunately, a simple idea makes it possible to overcome this stulifying complexity. The idea is a direct extension of Ohlin's work, and consists in formalizing the notion that trade in goods is a surrogate, within certain limits, for trade directly in the services of primary factors of production. For this notion says in effect that commodities entering into final consumption are themselves intermediaries between the ultimate factors of production and satisfaction, utility, welfare, gratification, or enjoyment. It follows that the utility function can be defined directly with the amounts of the primary factors' services as its

arguments:

$$W = W(d) = T(f),$$

where W is the level of utility or welfare, d is the vector of commodities, and f the vector of primary factors. A level curve of the function T will be an indifference curve in the services of the primary factors.

Different individuals, firms, organizations, classes, groups, communities, or nations possess the primary factors in different relative amounts, but the supply of any particular factor can be treated as fixed in amount at any moment of time. This is because leisure activities, like productive activities, require factors too and because there are only 24 hours in the day. A man never withdraws his land, labor, or capital from his utility function, though he may withdraw parts of them from the market. Thus his exchange of these factors can be treated in the simplest type of Edgeworth box diagram, using the indifference curves defined for factor services. Equilibrium terms of trade will give the relative prices of factors directly, from which can be computed the prices of whatever commodities are produced in the equilibrium.

The notion of the factor service indifference curve does not require, strictly speaking, any special assumptions about homogeneity, joint production, or externality. This is because all of these things are only complications in the general transformation of factor services into enjoyment. In the present exposition, however, it will be convenient to assume that if a commodity can

be produced by more than one production function, each of them produces it alone. Some single activities or production functions may produce more than one commodity, however. If so, it will be assumed that they appear in fixed proportions, so that they can be treated effectively as a single composite commodity.[1] It will also be convenient to assume that at least as many goods are produced somewhere in the world, by productive functions homogeneous of the first degree, as there are primary factors. Beyond that, it is easy to include particular industries whose optimal plants more than satisfy world demand at the average cost of their products. Moreover, it will be unnecessary to assume that any activity or productive function continues to give constant returns even at very low levels of output.[2]

The question now is to see with how much impunity one can neglect goods and services in his trade analysis and work directly with the indifference curves in the services of primary factors. This can be answered only be deriving the factor service indifference curves for the situations to be analyzed.

It will be discovered that complete equivalence is possible only under completely free and costless trade, but that even under the most severe protection of factors, loci or zones of equivalence exist. In practice, then it will be necessary for trade, migration, and investment to observe these loci or zones if the

solution to whatever problem the protection solves is to remain undisturbed by the secondary consideration of the movement of goods versus that of factors. Let us begin, for clarity's sake, with the simplest cases first and gradually add generality.

To every point and slope at that point in the commodity space, corresponds at least one point and a slope in the factor space. One can thus trace his way around an indifference curve in the commodity space, and pick out the corresponding indifference curve in the factor space. This follows directly from the two definitions of income:

$$Y = p'd = p_o'f \quad ,$$

for the vector d is a point in commodity space, and the vector f is one in the factor space. The vector p is a vector normal to the price plane which defines national income at market prices, and which is tangent to an indifference curve. Similarly, the vector p_o is normal to the factor-price plan which defines national income at factor cost, and which must be tangent therefore to an indifference curve in factor space.

It is usually very difficult to go from commodity prices back to factor prices. Under linear programing assumptions, of course, it can be done feasibly by solving the dual to the problem: Maximize p'd subject to $Gd \leq f$. Figure 1 shows a much simpler, though equivalent, method, which is particularly helpful if substitution among the various factors is possible. The Figure shows a mapping from the commodity space, placed in the third

quadrant. The curve I-I is an ordinary indifference curve, and it is shown tangent to the line q_1q_2 at point d. The isoquant Q_1, in the first quadrant, is for the amount of the first commodity (bread) that is worth the entire income corresponding to the tangent to the indifference curve at point d, or the amount q_1. This is equal to Y/p_1. Similarly, Q_2 shows the combinations of the two factors, labor and machinery, that can produce an amount of the second product (machines) worth the entire national income at the prices holding at point d, or the amount q_2. This is equal to the Y/p_2. The isoquants have not been drawn as smooth curves, but rather as segments connecting distinct activities for producing each product. Smooth isoquants are the limiting case, where one knows an infinite number of processes for producing an item.

The factor prices can be found directly now simply by finding the line, 1-2, in the figure, supporting both isoquants. This line immediately gives thevectors, 1 and 2, or the inputs of factors into the two products. Thus from commodity prices alone, we derive the prices of the factors and the coefficients of factor use.

The next task is to connect the point d on the commodity indifference curve with the proper point on the line 1-2. This can be done geometrically by forming the parallelogram $01_a(1_a+2_a)2_a$, where 1_a is the vector corresponding to d_1, the consumption of bread, and 2_a the vector corresponding to d_2, the final consumption

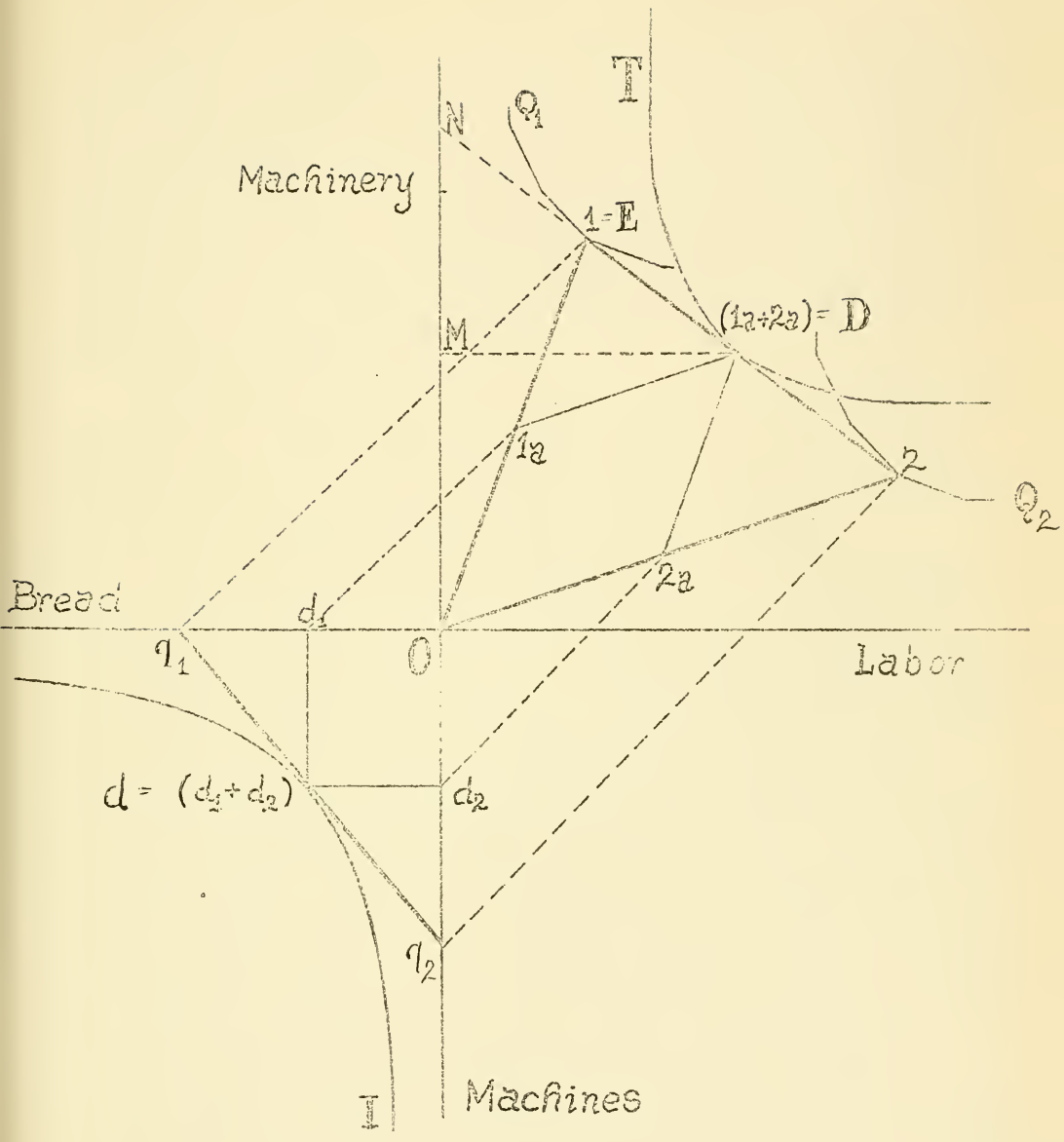
T
N
Q_1
Machinery
1=E
(1a+2a)= D
M
1a
2
Q_2
2a
Bread
d_1
η_1
0
Labor
$d = (d_1 + d_2)$
d_2
η_2
I
Machines

FIGURE 1

of machines. The point D, or $(1_a + 2_a)$, the sum of these vectors, is a point corresponding to d. As d is allowed to shift, following the indifference curve I-I, as the price place rotates to remain tangent, as the factor-price plane, 1-2, also rotates, and as the vectors 1 and 2 shift, other points, analogous to the point $(1_a + 2_a)$ depicted, will be determined and they will trace out the factor-service indifference curve, T-T.[3]

Algebraically, the point $(1_a + 2_a)$ can be found directly from the vector of final demands, d, and the matrix, G, of the input coefficients, g_{kj}: $f = G\,d$, where f is the vector of the amounts of the factors whose services enter into final demand.

Figure 1 also shows the simplest application of the factor-service indifference curve to international trade, migration, and investment. Suppose point 1, or E is the endowment point; it represents the amounts of the two factors which the country in question possesses. Under free trade in the services of factors, the country would presumably trade from E to the point D on the highest attainable curve, T-T, where the trading line was tangent to that curve. Since the point E lies entirely within the cone, OE2, formed by the input vectors, commodity trade alone could discharge that entire exchange in this case. Since E lies on the vector 1, however, the country would specialize on the first commodity (bread), and thus trade from the point q_1, on the bread axis of the commodity space. It would, of course, trade to the point of tangency, d, of the trading line with the curve I-I corresponding to the

curve T-T reached in the factor space.

Suppose now that the country applies a tariff to imports from the rest of the world and simultaneously raises a tax on imports of labor services and on exports of those of capital. These taxes themselves dictate what must be the tariffs on commodities, so one can think of the problem in terms only of the taxes on the services of factors, whether they are directly exchanged or exchanged wholly or partly through commodities.

This is shown in Figure 2, where T-T and I-I are the same curves as in the first figure. A foreign offer curve in the factor space can be derived in the ordinary manner from the other country's T-curves, and it is shown as passing through E, the endowment point, D, and R in the figure. If the country applies optimal factoral protection, the foreign offer curve will be tangent to the highest attainable domestic T-curve, as shown in the figure by the tangency point D. The slope of this tangent, ND, gives the ratio of domestic factor prices, and they determine the input coefficients. The vectors 1 and 2 show the ratio of machinery to labor in each industry, sections of whose isoquants also have been drawn through the points 1 and 2 The line 1-2, tangent to the two isoquants, is parallel to the line ND.

The endowment point, E, has been taken to lie on vector 1, which means that under free trade, or under factoral protection any less severe than that represented by the slope of the line 1-2, the country would be superspecialized, for E would lie outside the production cone, 012.

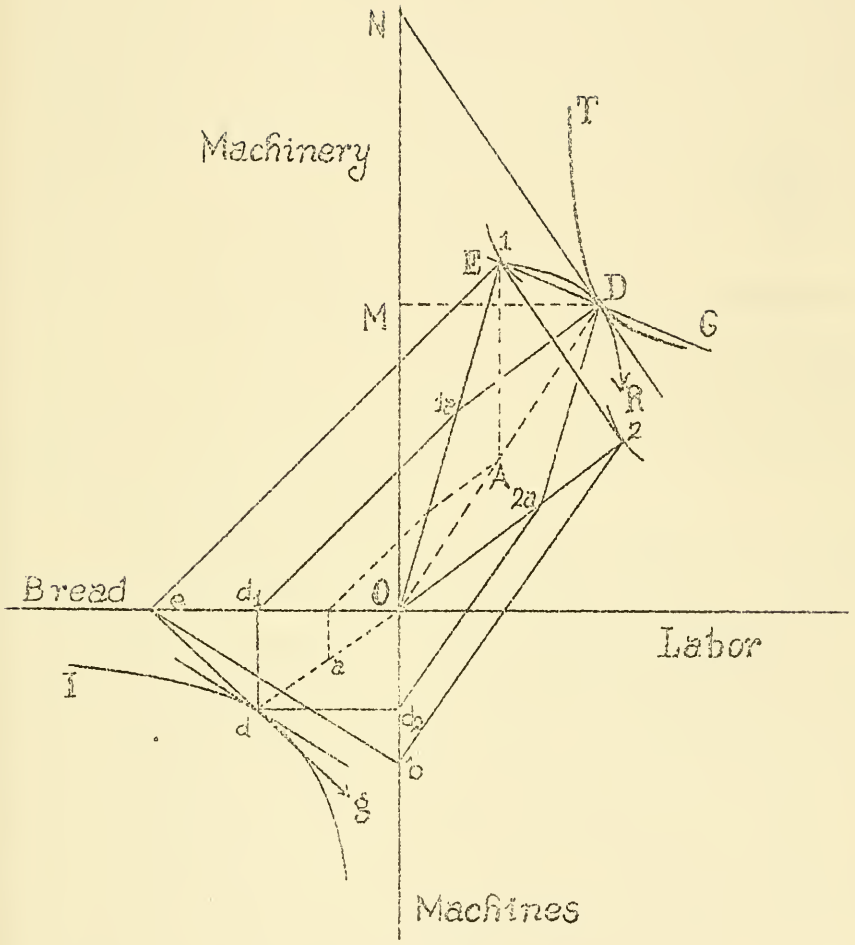
N
Machinery
T
E
1
D
M
G
1a
R
2
A
2a
Bread
c
d1
O
Labor
I
a
d
d2
b
g
Machines

FIGURE 2

The country can trade factoral services from E to D in several equivalent ways, subject to a qualification which must be introduced later. First, it may specialize at E on the production of the first commodity, produced at e on the bread axis of the commodity space, and trade bread for machines to the point d, which corresponds to the point D in the factor space. Point d is derived from D just as e is derived from E: line D l_a is parallel to vector 2, and line l_a d_1 is parallel to line 1 e. The point d_1 is the bread-coordinate of the vector of final consumption, d. Thus the line 1DG maps into the line ed, whose slope gives the ratio of goods prices corresponding to the ratio of factor prices, read off from line 1DG, obtaining in the other country as a result of the factoral protection.

The country may also hire foreign workers and rent out its machines, paying entirely thereby for the labor, until point D is reached. Commodity production will then take place at d and there will be no exchange of commodities, except in discharge of factor payments. It must be assumed, of course, that nationals of either country purchase at the relative prices of their home country, wherever they in fact may work or live. Thus, in this analysis, a country is only a regime of relative prices which pertains to a group of well-defined nationals, and it does not necessarily correspond to national boundaries.

In both of the preceding examples, both commodity trade and direct exchange of factors were taken as separately balanced.

Intermediate combinations of commodity trade and factoral exchange, represented by any point intermediate between E and D and lying on the line ED would share this property. The property is unessential however, for the only global constraint is that no consumption point, like D, can lie above the line EG, extended, for then the value of national consumption, at world prices, would exceed national income at world prices. Thus the formula for that the line EDG, is the national income identity

$$p_{o1}^{B} D_1 + p_{o2}^{B} D_2 = Y^{A} \quad ,$$

where the superscript A refers to our country, B to theirs, and where Y is national income.

Instead of balancing the current and the capital-labor accounts separately, the country could, say, export a surplus of capital services and receive a surplus of goods on current account. Point A represents, for example, the case in which the country hires directly no foreign workers, but places abroad EA units of its machinery. It produces at point a in the commodity space, which has been made to lie on the ray Oad, which corresponds to the ray OAD in factor space. By placing point A on OAD, we illustrate the special case in which all commodity trade is the interest on foreign capital, or rather the rental on domestically owned machines operating abroad. The country imports, but does not export commodities, and it imports some of both. Other examples of unbalanced current accounts could be illustrated by placing

A anywhere within the cone OlG, except on the line segment 1DG itself.

All such points would be perfectly equivalent only under free trade, however. "Equivalent" here means that any point, or assignment of factors to the two countries, produces exactly the same total output of each commodity. The output is then assigned to the two countries according to the consumption point, D. Under free trade, this point will lie where factor-service indifference curves of the two countries are tangent. Only under free trade will the input coefficients be the same for each product in each country, and this condition is necessary to ensure equivalence.

Figure 3 illustrates the difficulties which arise when the input coefficients differ, as under protection. This figure is admittedly quite difficult to follow, but I see no other way to handle the very complicated problem that it resolves. The conclusion to the analysis which it permits one to make is fairly striking and at variance with existing notions on the relationship between trade and factor movements. A certain amount of attention to the figure will repay amply the reader interested in this subject, for it would require much more time and effort to understand it by other methods. As an aid, the figure has been presented also with the constructional scaffolding removed to illustrate the main conclusion more clearly.

In the main, or bottom, diagram of Figure 3, T^A is one of country A's factor-service indifference curves, and T^B one of

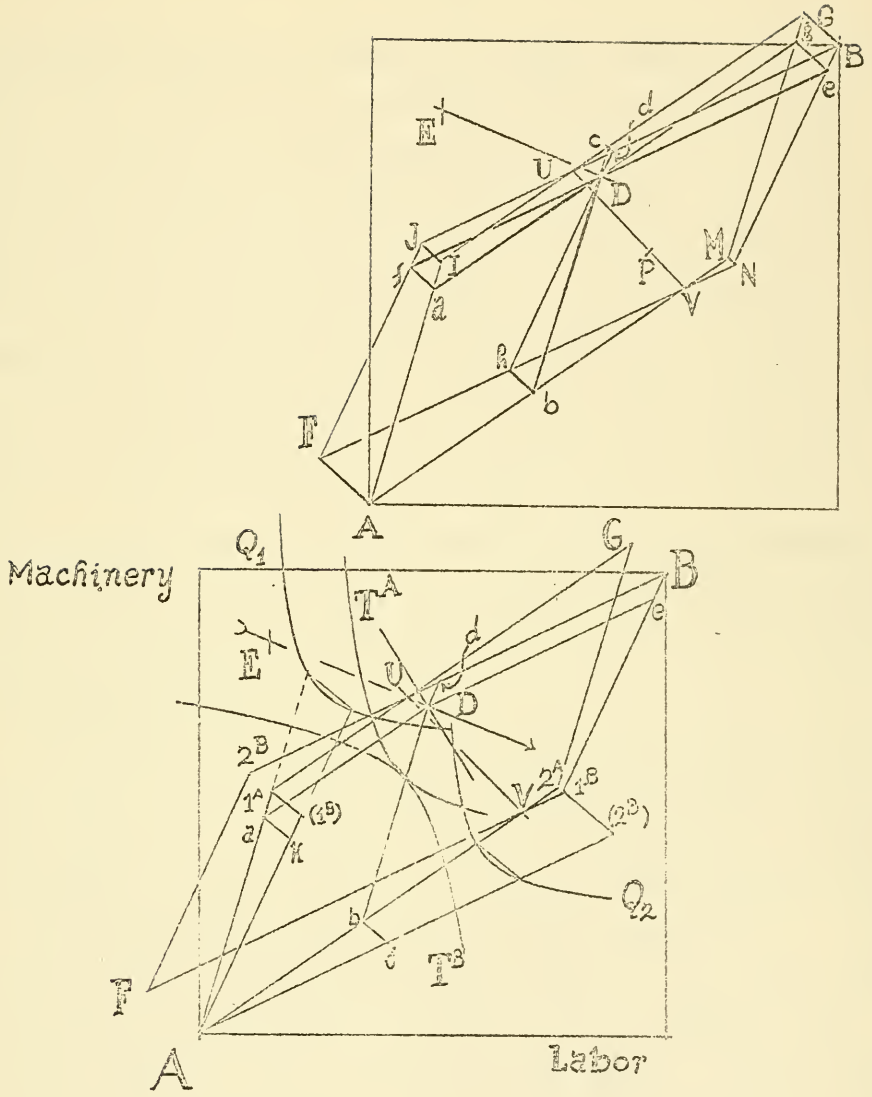

FIGURE 3

country B's, referred to country B's origin, point B. The endowment point is E. Assume that only country A applies protection and raises the terms of trade in her favor in such a way that the trading line becomes ED (shown as an arrow). The slope of this line gives the ratio of factor prices in country B, as well as the slope of her T-curve passing through D. Two representative isoquants, Q_1 and Q_2, have been drawn, referred to A's origin, A, for the first and second commodity. The vector (1^B), extended, intersects Q_1 where that isoquant has the same slope. These vectors show the factor proportions in country B for the two products, and can be compared with vectors 1^A and 2^A, which give the proportions for country A. They intersect the isoquants where they have the slope of the factor-service, or T-curve, passing through D and pertaining to country A.

The actual points 1^A, 2^A, (1^B), and (2^B) are found as follows. The point D, referred to country A's origin, corresponds to a given point in the consumption space, as we have seen in discussing figure 2. That point tells how much of each commodity country A would produce and consume were her factor endowments at D. The amounts of the two commodities which country B would produce and consume with the endowment of factors that D, referred to her origin, point B, can be found in the same manner. Vectors a and b show the factor requirements, under A's input coefficients, of A's consumption of the first and second product; vectors e and d do

the same thing for country B. The amounts of each commodity produced in both countries can be added and we can ask the following question: how much of the factors would country A need, with the input coefficients already determined, to produce that joint world output? The point 1^A shows how much would be needed for the first, and the point 2^A how much would be needed for the second commodity. The sum of these vectors, given by the point G, shows the total requirements of the two factors. Notice that point G lies outside the Edgeworth-Bowley box, AB, of actual world endowments.

In exactly the same way, but operating from origin B, we can derive the point F, which shows the amounts of the factors which country B, with her input coefficients, would require to produce the same world output of the two commodities. The point F is the sum of the vectors 1^B and 2^B, which, in turn, are the same as the vectors (1^B) and (2^B), except that they are referred to origin B rather than A.

The parallelogram, $A1^AG2^A$, is, in effect, an equalization region; it is the equalization region which would hold if there were another country with country A's factor prices and if the Edgeworth-Bowley box pertaining to that country and to country A were given by the points A and G. Within the equalization region, AG, any point, P, distributing factors between the two countries could assume any position and equivalence would be maintained.

But country B's equalization region, BF, does not jibe with country A's, and so a promiscuously wandering point P would alter the actual amounts of the two commodities produced by the two countries, and make world production inconsistent with world consumption, given by the point D when referred to the appropriate origins.

The difficulty can easily be generalized to any number of factors and commodities. The line ed in Figure 2 is related to the line ED as follows:

$$p^* = G' \, p_o^B \quad .$$

where G is the matrix of country A's input coefficients. The vector p* is normal to the line ed, or to its n-dimensional hyperplanal equivalent. Its elements are not necessarily country B's prices, for these are given by $p^B = (G^B)' \, p_o^B$; that is, they are determined under B's input coefficients. Thus country A may perceive a difference between exchanging factors directly and producing from point D and not doing so, but producing from point E instead. The latter may permit country A to exploit a favorable difference between the slope of p^B and that of p^*.[4] Unbalancing current payments further complicates things.

The use of the factor-service indifference curve requires equivalence, of course, in order to suppress the distinction between goods and factors. Fortunately equivalence still exists, but its domain is reduced.

The upper part of the diagram has been clarified to show the domain of indeterminacy more clearly. It can be seen that the two

equalization regions special to each country seem to intersect each other in the line segment UV. This line segment is the locus of indeterminacy in this two-dimensional case: a point P which shows how much of each factor actually works in each country can be placed anywhere along that line and total world output of each product will be the same as if P were at D, the point showing how much of each factor each country consumes under autarky.

That any point on UV produces the world consumption of the two commodities, or produces D, can be proved as follows. Extend eD to f, on JF, aD to g on GM, dD to h on FN, and bD to c on GI. (The corners I and M of country A's equalization region and J and N of country B's have been relabelled for easier reference.) Now a I, equal to gG, gives the amount of resources which country A, with its input combinations, would devote to producing country B's consumption of the first product. Country B would devote Be or Jf units of the factors to the same production.[5] Therefore, since vectors AI and FJ give the same total production of the first product and so do Aa and Ff,

$$\frac{Aa}{aI} = \frac{Ff}{fJ} \quad \text{and}$$

$$\frac{Mg}{gG} = \frac{Ne}{eB}$$

which means that the lines FA, fa, JI, GB, ge, and MN are all parallel to one another. But ag is parallel to IG and fe to JB.

Therefore

$$\frac{JU}{\overline{IU}} = \frac{UB}{\overline{UG}} ,$$

by similar triangles. This means that if P coincides with U, say, country A will produce all of the world's first commodity and part of its second. Country B will specialize on the second commodity, and IU units of factors at country A's input coefficients, together with BU units at country B's, will produce the same amount of the second commodity as AM units at country A's input coefficients, or BJ units at country B's. Similarly, cd and hb are both parallel to GB, etc., so that

$$\frac{FV}{\overline{AV}} = \frac{VN}{\overline{VM}} ,$$

which means that placing P at V will also enable the two countries jointly to produce the same total world output of both commodities. It is easy to show that any convex combination of points U and V, that is, any point on the line segment UV, will do the same thing so that any point P on that segment produces D.

Now, as the figure is drawn, no point on UV also lies on ED, whose formula is $p_{o1}{}^{B}D_1{}^{A} + p_{o2}{}^{B}D_2{}^{A} = Y^{A}$, where D_k is the amount of factor k which country A consumes and where Y^{A} is country A's income at world prices. Thus current payment <u>cannot</u> be balanced in this instance, although it is possible, under some configurations, for ED and UV to intersect and give rise to <u>one</u> point where both

payment accounts can be separately balanced. In general, then, a country applying protection does not want to balance its current account.[6]

2. The factor-service indifference curve in higher dimensions

Perhaps the greatest practical advantage of the factor-service indifference curve is that it is easy to visualize and work with in many dimensions, so that one can use any number of factors and commodities in his analysis. This is particularly important when dealing with intermediate commodities in international trade, for their usefulness, both practically and theoretically, is in ensuring the existence of equalization regions. How does the equalization region appear in higher dimensions?

Figure 4 depicts it in three dimensions. The world Edgeworth-Bowley box is the cube AB, and country A's equalization region is the parallelepiped A1 (1+2) (1+3) (1+2+3). The three basic vectors, 1, 2, and 3, of this parallelepiped refer to the three commodities, and the sides of the cube give the amounts of the three primary factors, labor, machinery, and fuel. Country B's equalization region, which has been omitted to prevent graphical confusion, has its origin at B and intersects country A's in a plane, corresponding to the line UV of Figure 3.

The volume of the equalization region, which tells something about the likelihood that factor prices can be equalized under free and costless trade, is simply the determinant of the matrix

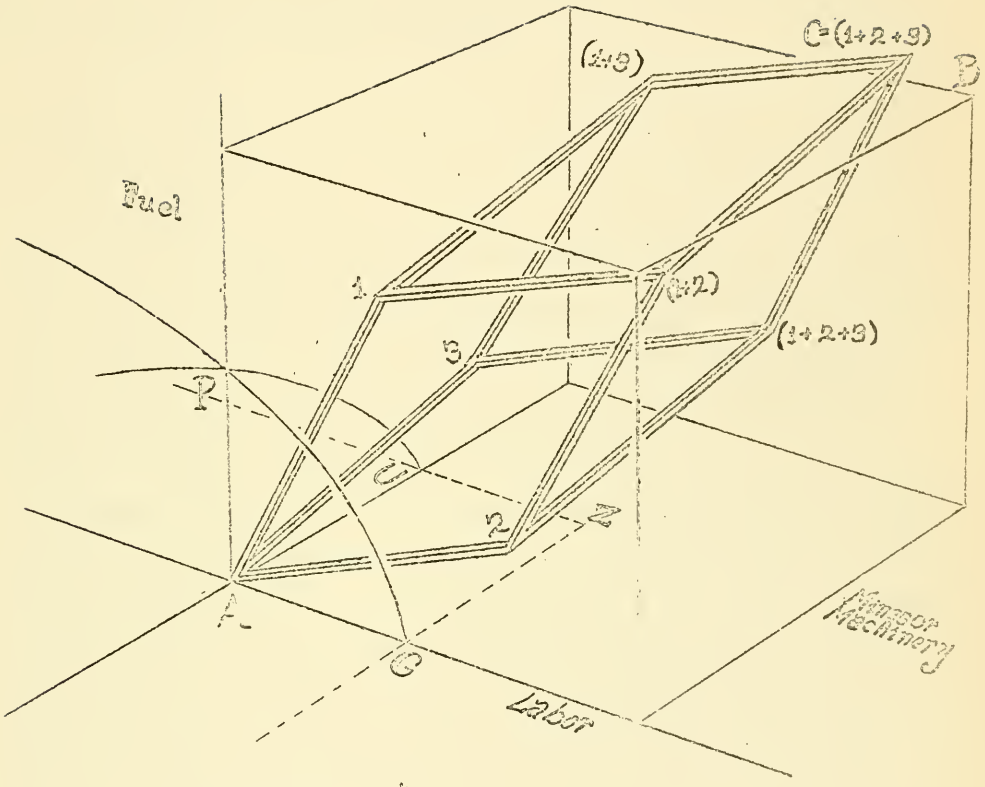

FIGURE 4

of three vectors, 1, 2, and 3, or in other words the determinant of the matrix $[F_{kj}]$, $\left|F_{kj}\right|$, where F_{kj} is the amount of the kth factor working directly in the jth industry. This determinant can be compared with the determinant

$$\begin{vmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{vmatrix} ,$$

where C_k is the kth coordinate of the point C [equal to (1+2+3)], which shows how much of each factor country A would use to produce world consumption, in order to assess the likelihood of factor price equalization under free trade. All of these results, of course, generalize directly into n-dimensions.

Country A's factor-service indifference curves would be bowl-shaped in three dimensions, and would be analogous hyper-surfaces in higher ones. Just as equalization regions can easily be drawn up for actual countries, using only input-output data, a simple consideration also makes it possible to draw up actual factor-service indifference curves. The factor-service indifference curve is nothing but the isoquant of the aggregate production function, many examples of which have already been estimated and fitted.

3. The factor-service indifference curve when there are more goods than factors.

The case in which there are more goods than factors is important

because it undoubtedly is the one which holds in the world, no matter how finely one defines factors. If each type of machine is considered a factor, it is also a commodity. The same is true of each type of skill acquired through training; the training itself is a commodity. Meanwhile, thousands and thousands of classes of products are distinguished by various government bureaus and, notably, in the tariff. I should guess that there are about 100 million different commodities produced each year by about 700,000 different types of factors.

Moreover, it is useful to think of production functions separately from products in many cases because which factors are used will often vary from country to country. The same sewing machine may be produced by quite different machinery in the United States and India, say, and for many purposes it is useful to regard the machinery, rather than "capital", as primary factors. Similarly, land in the Gangetic plain is different from land in the American southwest and it would call for a different type of cultivation even if wages and interest rates were identical in the two locations. This would be true even if they both produced exactly the same variety of wheat. There is no reason not to posit entirely different production functions whenever their arguments differ as in these examples. As long as the number of products, to be traded one for another, so greatly exceeds the number of factors even when they are so defined, there is no danger of running out of products, but there is great convenience and realism in not having to give

metaphysical interpretations to any factor such as land or capital.

The simplest case in which commodities are more numerous than factors can be shown graphically. The graph in turn can illustrate another important characteristic of all cases in which commodities outnumber factors, namely the indeterminancy of output and the determinacy of n - r prices, where r is the number of factors, once r prices (or r-1 relative prices) are given. In Figure 5, which is drawn in perspective, the factor space is on the horizontal plane. Three isoquants, Q_1, Q_2, and Q_3, are drawn in that space. These three isoquants are the ones which correspond, respectively, to the amount of the first product, bread, indicated by the point a on the bread axis of the three-dimensional commodity space, the number of machines, indicated by point b, and the amount of fuel, indicated by the point c. The straight lines a1, b2, and c3 indicate the mapping from the commodity space into the factor-space, and lines parallel to them, going from the appropriate points along 0a, 0b, or 0c to the vectors 1, 2, or 3, permit one to go from any point on the plane abc to the line 1-2, and thus, for instance, to obtain from a tangency of abc with an indifference surface a corresponding point on the factor-service indifference curve.

Suppose now that the price of fuel should fall, so that point c were farther from 0 than shown in the diagram. A line parallel to c3 would strike an isoquant lying outside the cone 012, and that isoquant would be unattainable with any factor combination worth

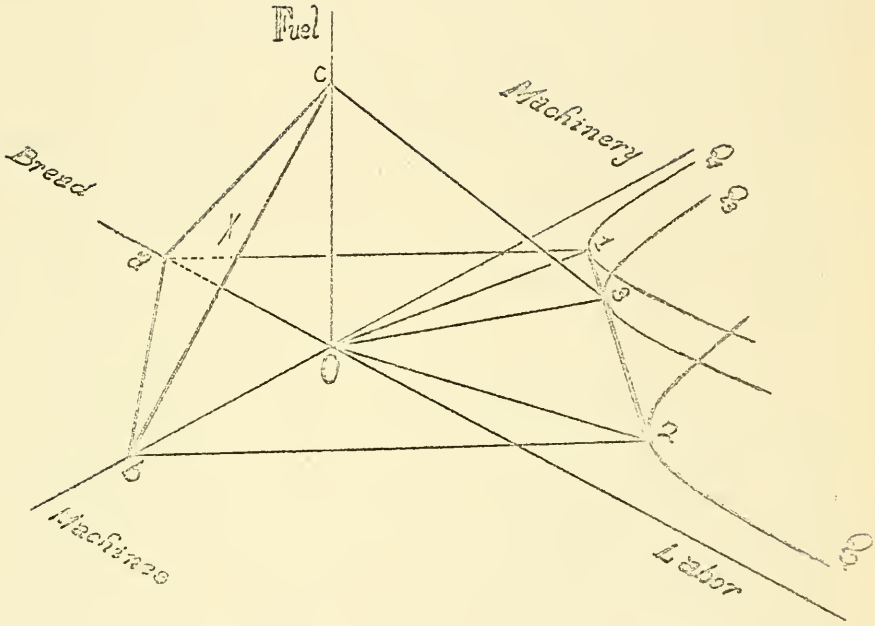

FIGURE 5

only as much as the combinations along 1-2. Fuel would therefore cease to be produced. If fuel became mor expensive, the point c, now nearer to 0 than depicted, would map into a closer isoquant than Q_3. The line supporting that new isoquant and, say, Q_1, would fail to reach Q_2; hence the price of machines would have to fall along with the price of fuel if the price of bread stayed the same. In general, then, the prices of any two commodities, by determining the prices of the two factors, which prices are all that determines any commodity's prices, determine the price of the third (or the fourth, fifth, etc.) commodity. This is why the concept of the equilibrium tariff is so important when the number of goods exceeds the number of factors.

4. Intermediate commodities in international trade

We are finally in a position to deal with intermediate commodities in trade. It is easy to see why they necessarily have been omitted, for the most part, from discussions which assume only two commodities and factors, for their main importance is in increasing the effective number of commodities; only multi-commodity models therefore can handle all the problem they pose.

Because a given sector may disappear, domestically, for lack of sufficient protection, or never exist for lack of some resource specific to it, and because some products, intermediate or final may be untradable, it is necessary to distinguish among several cases and to be able to say when each will hold. Noncompetitive

intermediate goods pose a thornier problem than competitive ones. At the same time, they are even more important in actual trade. We shall begin, therefore, with the case of competitive or domestically produced, though perhaps untraded, intermediate products. The simplicty of this case derives from their failure to have any separate role in the determination of prices under it.[7] Whether or not the case holds, however, depends upon the size and disposition of the equalization regions of the trading partners, and that question must therefore be examined.

Again, we begin with the simplest depiction in order to strengthen intuitive understanding, which is the basis for creative generalization afterwards. Figure 6 shows a situation in which two goods are produced and traded, each of which is an input into the other. Each commodity is produced by a production function homogeneous in all four inputs, and this property enables one to construct isoquants, Q_1 and Q_2, for integrated production functions which use only the two primary inputs. The construction cannot be done geometrically, however, but requires the solution of the following sets of equations, which are defined, for later reference, in terms of any number of commodities and factors:

$$(1) \quad 1 = \Psi_j (f_j, a_j) \qquad (j = 1, \ldots, n) \quad ,$$

$$(2) \quad p_i = \Phi_{ij}(f_j, a_j) \quad , \quad p_{ok} = \Phi_{kj} (f_j, a_j)$$

$$(i = 1, \ldots, n; \quad k = 1, \ldots, r) \; .$$

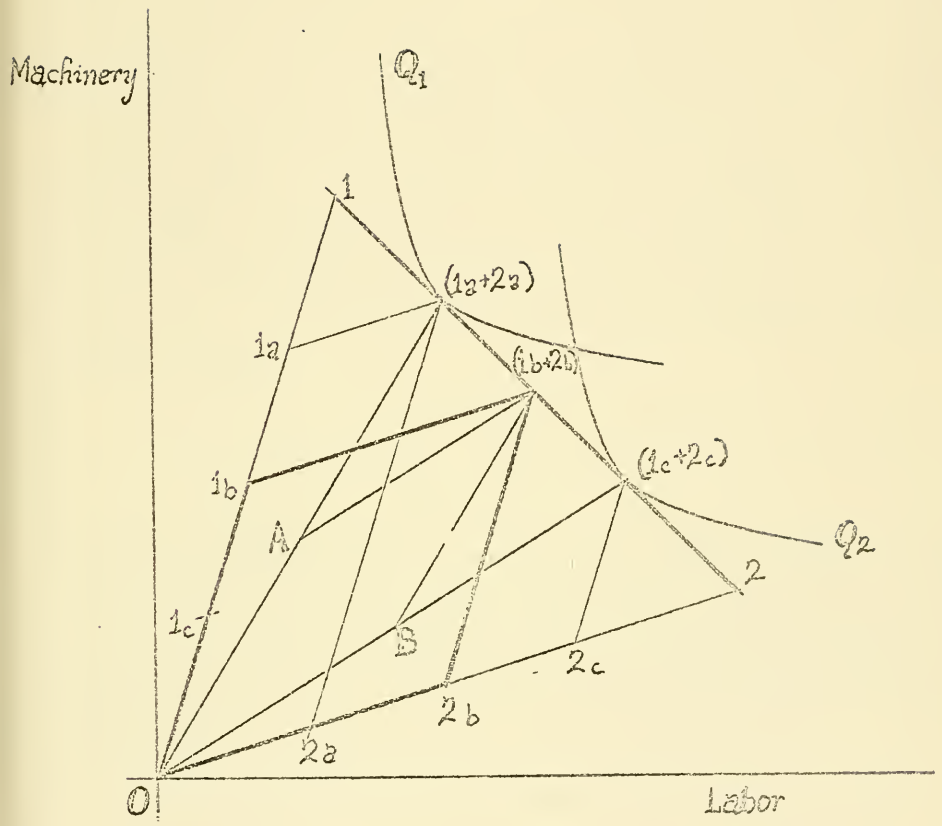

FIGURE 6

There are $n^2 + n + nr$ of these equations, and n^2 coefficients a_{ij} and nr coefficients f_{kj} for them to determine, in addition to n prices of the commodities, final and intermediate, and r prices of the factors. Thus r equations are lacking, or rather r - 1 since relative prices are sufficient to determine all the input coefficients. These r - 1 relative prices can be thought of as those of the primary factors themselves, so that, given them, and given the above equations, all the input coefficients can be found. In particular, for two commodity example, the equations will give the Leontief matrix

$$(I - A) = \begin{bmatrix} 1 & -a_{12} \\ -a_{21} & 1 \end{bmatrix}$$

for every set of relative prices of the primary factors. It will also provide the direct input coefficients, f_{kj}, for each sector, and this information is sufficient to determine the amount of each primary factor used directly and indirectly per unit of final output.

This information in turn permits us to draw the final isoquants, Q_1 and Q_2, in Figure 6. Suppose first that there is final demand for only the first product. Some of the first product will go into the second which in turn will be used to make the first. The amount can be found by solving the system

$$\begin{bmatrix} f_1 \\ \\ f_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ \\ f_{21} & f_{22} \end{bmatrix} (I - A)^{-1} \begin{bmatrix} q_1 \\ \\ 0 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} x_1^* \\ \\ x_2^* \end{bmatrix}$$

where x_j^* are the gross domestic outputs when the final demand, q_2, for the second product is zero. Since the x_j^*, $j = 1,2$, are known, the $f_{kj}x_j^*$ are also known, and we can plot the amounts of factors directly employed by the two industries as in Figure 6. Thus the vector 1_a shows how much of either factor the first industry employs, when only the first product is to be consumed finally, and when factors are available according to the budget line 1-2. The slope of that line, as we have seen, determines the input coefficients, which therefore will change if its slope changes. The sum of the vectors 1_a and 2_a gives a point, $(1_a + 2_a)$, on the isoquant, Q_1, of final output of the first commodity.

Now suppose that only the second product is demanded, and that the allocation of primary factors to the two sectors in that case is given by the vectors 1_c and 2_c. Their sum is a point, $(1_c + 2_c)$, on the isoquant, Q_2, of final output of the second commodity.

The outputs corresponding to the two isoquants, Q_1 and Q_2, are the maximum amounts of the corresponding commodities which the country can buy with the income represented by the budget line 1-2. They correspond to points like a, b, and c on the commodity

axes of Figure 5, and are the points where the price plane, in the commodity space, intersects the axes of that space.

Any point on the price plane in the commodity space can be represented as a convex combination of the amounts q_1 and q_2. Since the production functions are homogeneous of the first degree and the coefficients remain fixed for any set of relative prices, the convex combination of outputs will involve a convex combination, represented by a point on the line 1-2 lying between $(1_a + 2_a)$ and $(1_c + 2_c)$ of their ultimate inputs of the two primary factors as well.

In Figure 6, the vector $(1_a + 2_a)$ shows the ratio of direct-and-indirect requirements of the two factors for the first commodity. The vector $(1_c + 2_c)$ does the same for the second commodity. Suppose that people in fact want to consume $\frac{A0}{0(1_a + 2_a)}$ units of the first product. They then can consume 0b units of the second. This amount is derived by drawing $A(1_b + 2_b)$ parallel to the vector $(1_c + 2_c)$ to the point $(1_b + 2_b)$ on the line 1-2, and then drawing the line $(1_b + 2_b)$ parallel to 0A to B on the vector $0(1_c + 2_c)$. As the relative proportions in which the two commodities enter into final demand shift, the point $(1_b + 2_b)$ also shifts, but remains between the two points $(1_a + 2_b)$ and $(1_c + 2_c)$.

At each such point there is a different equalization region. In one extreme case, where only the first commodity enters final demand, the equalization region is $01_a\ (1_a + 2_a)2_a$. At the point

1_a, the country produces nothing of the second commodity, which it imports as an intermediate into its only sector. The other country produces 2_a units of the second commodity, which it exports in return for the first. If someone passed a law ruling out trade in either commodity when it was to be used as an intermediate, there would be no equalization region at all if only one commodity were final. Factor endowments would have to lie on the vector $0(1_a + 2_a)$, and Q_1 would be the factor service indifference curve.

If consumption of the two commodities is at the point $(1_b + 2_b)$, however, the equalization region is $01_b(1_b + 2_b)2_b$. At 1_b, the country would specialize in the first, and the other country in the second, commodity. If the ukase against using traded goods as intermediates were repeated, the equalization region would shrink to $0A(1_b + 2_b)B$. The role of intermediates in international trade, under the set of assumptions used throughout this paper, is only, therefore, to determine the size of the equalization region.[8] The size of the relevant equalization region is found, it must be emphasized by using the matrix, F, of direct input coefficients, rather than G, that of direct-plus-indirect coefficients, which would yield only a smaller region lying entirely within the appropriate equalization region.

5. Untraded intermediate commodities

The problem of untraded intermediate commodities arises in two important contexts in international trade, even though it is

only an artificial problem under the assumptions made here, and under those in fact made by nearly all other writers as well. One context is that of exchange rate adjustment, where it is assumed that sectors producing untraded commodities will fare differently from those competing directly with import or vying directly for export markets. The other is in planning, where it is necessary to provide water, power, etc., for whatever industries are chosen to satisfy final demand and exports.

The relative importance of the inputs from any given basic industry varies according to the final industry in question, of course, so there is a problem, often neglected, of choosing those industries whose indirect requirements of labor and capital, etc., employed in industries producing untradables, fit the general scarcity or abundance of these primary factors. Thus, if electricity is capital intensive and aluminum is electricity intensive, it is better, where labor is cheap, to produce aluminum products, but to import the unfinished aluminum and thus to import cheap foreign electricity, indirectly.

The problem can be worked out by means of the concept of the equalization region. Figure 7 refers to a situation in which there are three commodities, all entering into each of the three sectors as intermediate inputs. The vectors 1_a, 2_a, and 3_a show the amounts of factors country A would devote to world production of the three commodities, 1, 2, and 3. The amount 1_a, for instance, also requires certain inputs of the other two commodities, as do 2

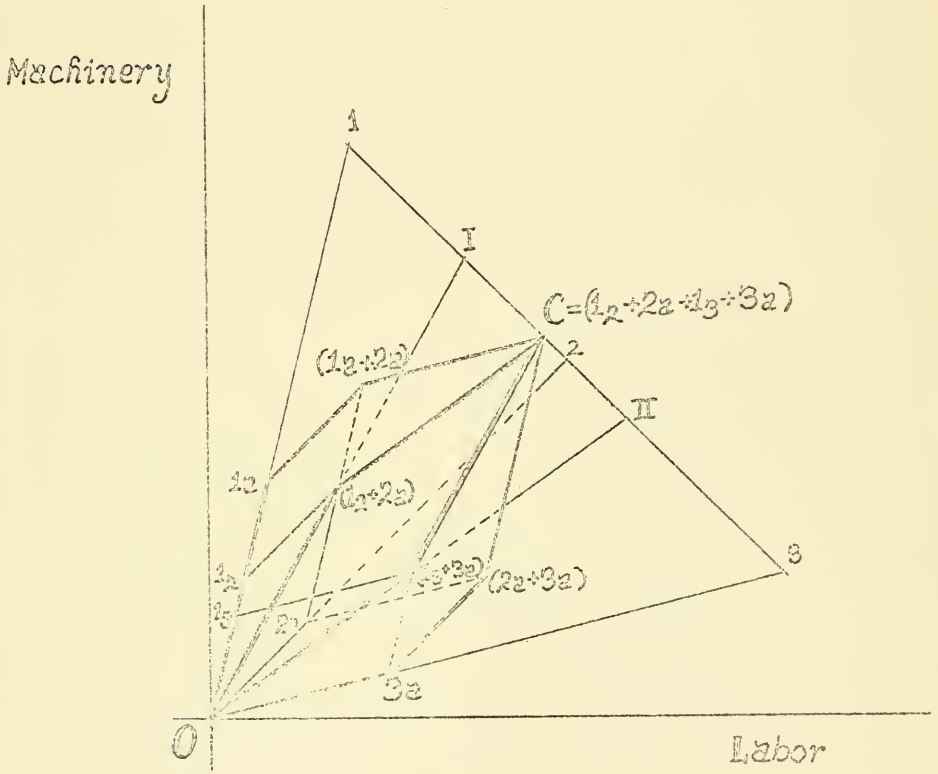
Machinery
1
I
C=(1a+2a+1b+3a)
(1a+2a)
2
II
1a
(1a+2a)
3
1a
1b
2a
(2a+3a)
3a
O
Labor

FIGURE 7

and 3_a. So long, however, as all three commodities are traded under their equilibrium tariffs, these inputs do not need to be distinguished in drawing up the equalization region 01_a $(1_a + 2_a)$, C, $(2_a + 3_a)$ 3_a. The six sides of this polygon correspond to the three vectors, just as though the parallelepiped, AC, of Figure 4 were projected onto the floor (labor-machinery space) of the cube AB in that diagram. All the intermediate relationships are already implicit in the direct input coefficients, which give the three basis vectors.

Now suppose that the first product cannot move, but must be provided in conjunction with the production of either of the other commodities. This immobility imposes two constraints. In the first place, even though the first product happens to be intensive in machinery, a country rich therein could not hope to produce the world output, 1_a, unless it produced the world output of everything. Thus the value of the first industry in carrying out the surplus services of abundant machinery is circumscribed. In the second place, some of it must be produced even by a country with abundant labor, which therefore would like to specialize on the third commodity. It must dilute the labor-machine combination appropriate to that sector with that appropriate to the first sector, to an extent depending upon the actual importance of the input coefficient, a_{13}.

This modification is known from the solution of the system that gave us the coefficients in the first place, however, and has been indicated by the point 1_3 on the diagram. That vector shows how

much machinery and labor the first sector uses to produce the amount of the third product used by the amounts, 3_a, of machinery and labor directly producing world output of the third commodity.

The point $(1_3 + 3_a)$, or B, therefore reveals the effective direct factor inputs of the third product. Indirectly the third product also uses a certain amount of labor and machinery working in the second industry, which in turn uses some labor and machinery working in the first industry. But the services of those amounts of labor and machinery can be imported through the tradable second product, and do not have to work in the same country as the labor and machinery indicated by point B.

The second industry, in order to produce the world output of the second product, must employ in conjunction with its own machinery and labor, given at point 2_a, an amount in the first industry which the point 1_2 indicates. The second product thus directly requires the inputs of the primary factors which the point $(1_2 + 2_a)$, or A, indicates.

The true equalization region is therefore given by OACB in this case, where the first product is an untransportable intermediate commodity to the other two.

The best way to regard the first industry, for either of the questions posed at the outset of this section, is as though it did not exist. Just pretend that only two sectors exist, and that the first uses factors directly as indiated by the vector I, and the other as indicated by the vector II. Each of those sectors, to be

sure, uses some electricity, or whatever product corresponds to the first product in the example. That electricity should be regarded simply as part of the process for producing the products using it, and decisions about investing in it should be made accordingly.[9]

6. Untradable final products

The problem of untraded final products is more serious, but even less considered, than that of untraded intermediate products. By dealing with them explicitly, however, we are able to take care of the problem of variable factor supply directly, for leisure activities can be considered as ordinary productive activities (to the possible dismay of Adam Smith), whose products are untradable. Then the supply of any given factor at any given moment is determinate and fixed.

Untraded final commodities can be treated under the same system we have been using, namely that of the factor-service indifference curve and the equalization regions special to each point in it and its slope in the factor space. Suppose, with reference to Figure 8, that the third commodity is final and untradable. Country A's equalization region, if the third commodity were tradable, would be given by the polygon $01_a(1_a + 3_a)$ C $(2_a + 3_a)2_a$. As before, the basis bectors, 1_a, 2_a, and 3_a, show the proportions and amounts of the two factors which country A would devote to the world output of the three commodities if her factor prices were those indicated by the slope of the line 1-2. Suppose that under the same equilibrium

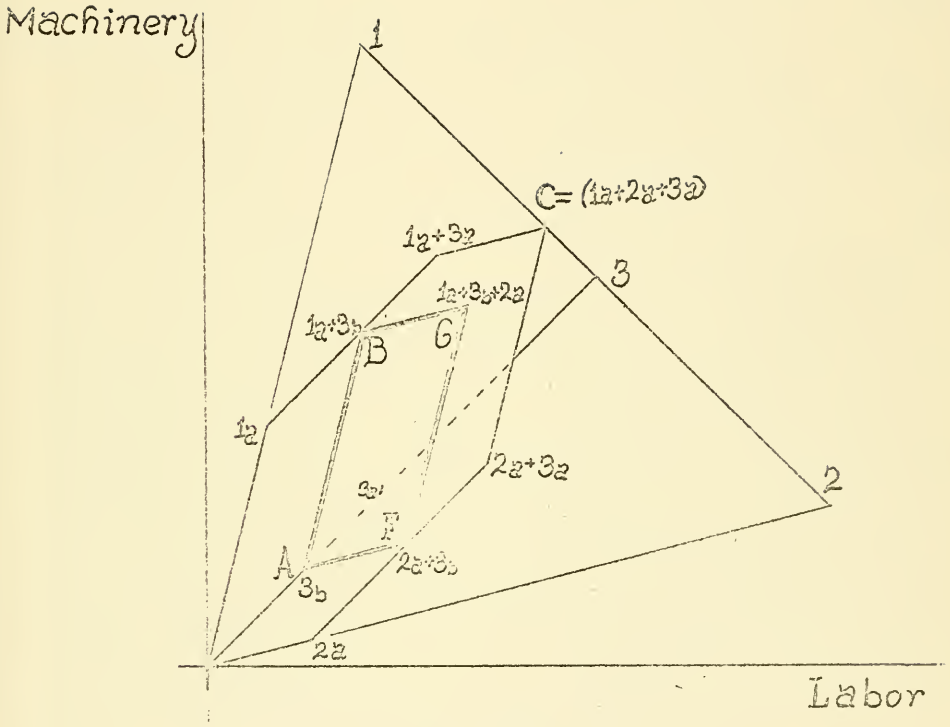
Machinery
1
C= (1a+2a+3a)
1a+3a
3
1a+3b+2a
1a+3b
B
G
1a
2a+3a
a
F
2
A
3b
2a+3b
2a
Labor

FIGURE 8

her own consumption of the untraded final commodity, measured by its input requirements, is given by the vector 3_b. This means that no matter what the actual working place of the factors, country A must employ at home the amounts shown by point 3_b, which point is therefore one corner of her effective equalization region.

The point G, or $(1_a + 3_b + 2_a)$, is another corner, because country B must consume and produce that portion of the world output of the third commodity which country A does not consume. But the first and second commodities are tradable, and so country A might produce the entire supply of the first commodity, at B, or $(1_a + 3_b)$, or the entire supply of the second commodity, at F, or $(2_a + 3_b)$, and, at either point, produce nothing of the other commodity. Thus country A's effective equalization region is ABGF. As we have seen, however, the actual point, P allocating factors to the two countries must lie on a certain locus transecting, as in Figure 3, the equalization regions of the two countries. Under free trade, of course, any point where the two equalization regions, which may now differ because of possible differences in the propensities to consume "leisure" in the two countries, intersect is a valid point allocating factors between the two countries. By the same token, as free trade is approached, strayings of the point allocating factors from its proper locus will affect the world general equilibrium less and less perceptibly.

7. The equalization region and indeterminacy when there are many, many products

As the number of products increases, the number of sides of

the polygon which is the equalization region increases in double measure. In actuality, therefore, its sides are almost perfectly smooth curves.

Another type of indeterminacy holds when there are more commodities than factors, even when the point allocating resources remains fixed on the locus corresponding to UV in Figure 3. A country can give up entirely the domestic production of such a product and shift its resources to the rest of its products without changing the previous equilibrium prices or consumption amounts of any factor or commodity in either country. This statement must be modified, however, if the reason for abandoning any industry is the failure to give it equilibrium protection, so that its product enters at a price lower than its domestic cost of production under the prevailing factoral protection. The product then becomes a noncompetitive import, properly so-called, and requires separate analysis. In addition, its disequilibrium tariff throws other tariffs, previously in equilibrium out of adjustment. A product using the cheapened import will now be able to supplant imports, or foreign production, if it is already exported. In the former case, it becomes an untraded product, intermediate or final, in the latter a noncompetitive import to the foreign country.

8. Noncompetitive imports

The most difficult problem of all in the application of the factor-service indifference curve arises in the case of noncompetitive

imports. Irving Kravis is, to my knowledge, the only writer who has given serious attention to them. He concludes that countries forsake almost all trade except in intrinsically noncompetitive products, those which they cannot produce domestically at any reasonable cost.[10] About half of American imports, for instance, are of noncompetitive articles, both final and intermediate. Some way of handling them rigorously therefore should be devised.

The conceptual problem involved is that a noncompetitive commodity can be considered only as a primary factor from the point of view of the country importing it, yet it is a final product from that of the country exporting it. This difficulty would be eliminated and the seriousness of one part of the problem largely dispelled, if we imagined that each country has at least a small amount of each type of primary resource, such as Dooars tea land, Champagne wine land, Bolivian tin land, etc., along with the appropriate climate in each case, so that one could observe how the United States, France, Japan, India, etc. would use their general factors, labor, capital, etc., to produce the respective specific products. If only iota of such factors were thus transplanted, it would make neglegible economic difference, but one could accurately assess the importance of the resources to the countries possessing them in sizable amounts. At the same time, the factors themselves, rather than their products, could be included in drawing up the factor-service indifference curves, which could then be placed in the same factor space.

This device would still leave open the possibility that a particular country might fail to protect a particular product, but import its entire requirements instead, as the United States imports all its manual typewriters, for instance. A more general treatment is therefore desirable. It is necessary that each country's factor-service indifference curve be placed in the same factor space, and so dimensions must be added to that space to include all the products noncompetitive as between any two or more countries included in the analysis. Thus even the exporting country will be thought of as using the noncompetitive item as a primary factor.

This is legitimate because there will be a one-to-one correspondence between the factor-service indifference curve in, say, the labor-mines space and that in the labor-fuel space, where fuel is a mined product. Refer again to Figure 4, and imagine that a factor-service indifference curve passes through the point Z, in the labor-mines space. Fuel is produced by labor and mines, but assume for now that all other commodities that enter directly or indirectly into final demand are produced by labor and fuel only. Mines do not enter into their production directly at all. Then labor and fuel can be regarded as their only primary inputs. To each point like Z in the labor-mine space will correspond one point in the labor-fuel space, and a curve in the former space will map into a corresponding one in the latter.

Now the analysis can proceed as before, except that the size of the labor-fuel space, which applies to both countries, is as yet

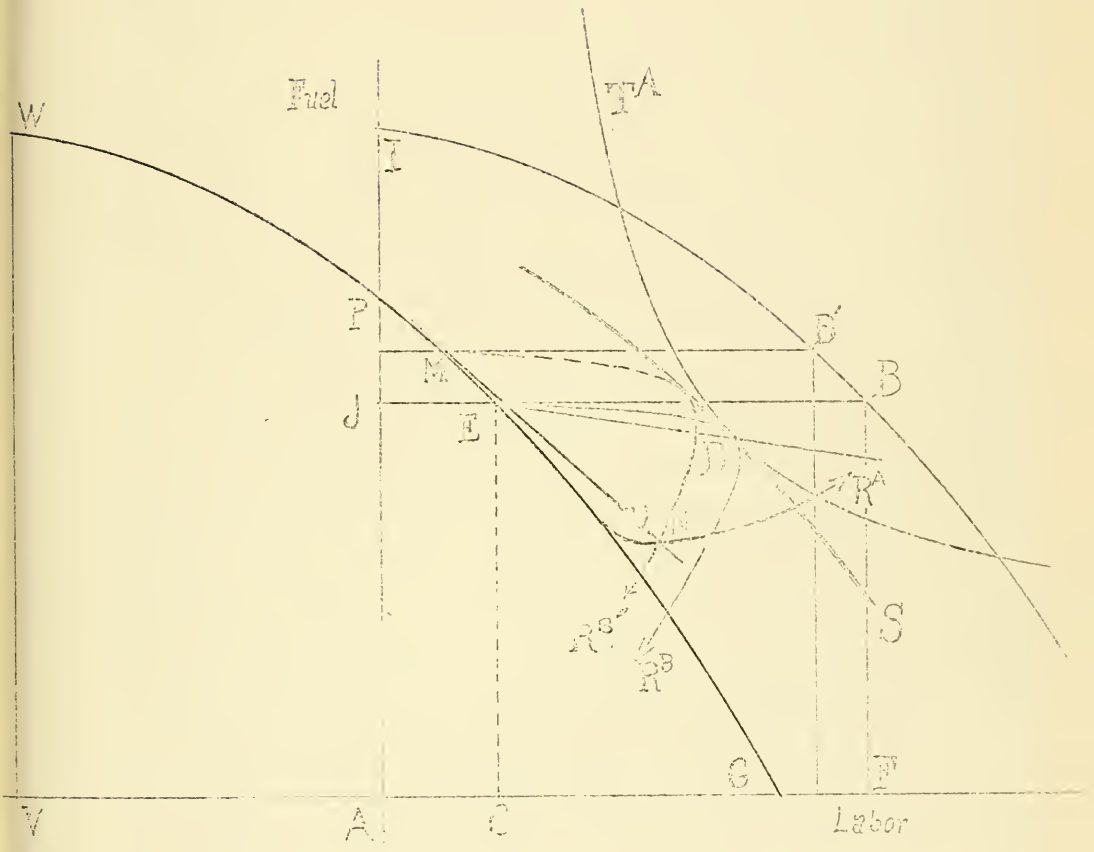

FIGURE 9

undetermined: it depends on the output of the country which actually possesses the fuel mines.

This can be determined, however, according to the same principles as those underlying the factor-service indifference curve when supplies of the primary factors are fixed and given. Refer now to Figure 9, which presents the essential argument in only two dimensions. In this example, fuel is produced with labor alone, but whatever labor is devoted thereto is, of course, unavailable for other commodities, which use fuel and labor as their inputs. The amount of labor devoted to fuel production is GC; Country A's total supply is AG, so A has AC units of labor available to combine with fuel for the production of enjoyment both directly and through the intermediation of various final and intermediate commodities. The production function for fuel is the curve GEPW. Maximum world output of fuel is equal to VW, where VG is the world's supply of labor. Country B is assumed to have CF units of labor, and nothing else, so the bottom side of the Edgeworth-Bowley box, for the fuel production shown, is AF. The vertical side is AJ, equal to EC. Country B's origin is at B, which must always lie on the locus, IB, which is GEPW shifted to the right, until W is at I.

The endowment point for this situation is the point E. Suppose that only country A applies protection, then it will presumably seek the point D, where the foreign offer curve, EDR^B, osculates its highest factor-service indifference curve, T^A. The slope of ED

gives the factor prices which now face the other country, but relative factor prices at home are given by the slope of T^A at the consumption point, D. The endowment point E will be an equilibrium endowment point if the slope of T^A at D is equal to the slope of the production function, GEPW, at E. The curve S is the envelope of the foreign offer curves derived from each point along GEPW, and its tangency with T^A tells where country A achieves its best tax on fuel exports.

Country A, of course, may pursue some other policy, in particular free trade. Her own offer curve is given by EUR^A. Under free trade, the intersection of this offer curve with R^B will determine the equilibrium terms of trade. This is shown to occur at N in Figure 9. The line MN, along which the two countries trade fuel and labor, is tangent to the production function for fuel at M, which becomes the new endowment point, replacing E. The origin for country B shifts to B' along BI, and country B's offer curve R^B shifts parallel to itself into $R^{B'}$, which intersects R^A at N. Thus the complication of noncompetitive goods can be handled within the same framework that we have been employing.

Figure 4 can now be used to visualize cases in which there are two primary resources besides the noncompetitive commodity. The production function for fuel has its origin at Z. The curve GP shows the output of fuel as the input of mines is kept constant at level ZG while the input of labor increases from G to Z and beyond. Similarly, UP is another slice of the production function, with the input of labor held constant at the level AG. These two

curves support a surface, like a bent piece of paper, which represents all the various possible outputs of fuel as functions of the inputs of labor and mines.

The same picture can be used to represent the case in which both countries have mines, or, alternatively, that in which fuel can be produced with labor and machinery, which are universally available. Suppose that in either case, however, country B fails to give fuel its equilibrium degree of protection so that it becomes a noncompetitive import.

The only complication this situation poses is that machinery or mining no longer disappears from the analysis, because it is specific only to fuel, but remains as a factor in terms of which, in addition to fuel, the factor-service indifference curve must be defined. Thus, in Figure 4, we must continue to use the three-dimensional, bowl-shaped isoquants of the original figure. In addition, the amounts of labor and machinery, or mines, must be those that are net of the amounts used up in one country to produce the fuel measuring the vertical height of the Edgeworth-Bowley box. But these are the only modifications that are necessary to the general analysis in this case.

The final case to be considered is that in which the non-competitive import uses other intermediates in addition to general primary factors. This case poses no new problem, however, for the intermediate goods can all be interiorized into a single direct

production function, using only primary factors, in the manner described in section 6 above.

Fortunately the practical application of this difficult analysis is easy to carry out. From the point of view of protection, the important question is: when do noncompetitive imports occur and how do they affect prices of other commodities? The first part of the question can be answered by computing each product's cost of production, under the factor prices given by the slope of the factor-service indifference curve, and seeing if that cost can be met under a given tariff. This calculation involves the formula

$$p = (I-A')\, F'\, p_o = G'\, p_o \quad .$$

The same calculation answers the second part of the question: one simply treats the noncompetitive imports as primary factors. This involves striking out their rows and columns from the Leontief matrix, (I - A), and their columns from the matrix of the coefficients of the direct inputs of the primary factors, F. Their prices are also eliminated from the vector p. The coefficients, a_{hj}, where the subscript h refers to the noncompetitive imports removed from the Leontief matrix, are added to the matrix F as rows, one for each noncompetitive import.[11] Thus we always can tell when each case: untradable inputs, non-competitive imports, and their various subcases, as well as all cases of specialization or nonspecialization in general, holds and how to treat it. We do not need a separate analysis for each case.

9. Capital and capital goods in international trade

Individuals, and groups acting like individuals, allocate their consumption through time in such a way as to maximize the satisfaction they derive from given present endowments and present and future opportunities to exchange both their assets and the products those assets make.

It is therefore possible to imagine an intertemporal indifference surface showing all the combinations in which present and future goods can be consumed that will give the same satisfaction when all prospects are viewed from the present. This surface will not be very useful directly in analyzing trade and investment through time, however, because in each time period during which saving and investment take place current demand will include that for investment goods as well as consumption goods. But an indifference surface, or hypersurface in many dimensions, can be defined easily to include investment goods, since they are included in the individuals' plans for consumption over time. As we have already seen an indifference curve in commodity space can be mapped into an indifference curve in factor space, and the same is true of the multi-dimensional indifference surface which includes investment goods in its arguments. A hypersurface in factor space corresponds to it, and can be used, in a manner analogous to the manner in which the orginary factor-service indifference curve has been used, to analyze trade, migration, and foreign investment over as many time periods and for as many separate goods and agents as desired.

It is useful to attempt to picture these indifference surfaces. Suppose that there are two commodities, bread and machinery, both of which enter current consumption. Machinery and labor produce bread and machines at the end of the current production period. Bread and machines are then consumed, and some machines withheld from consumption to be invested, as machinery, at the beginning of the time period immediately following. Let each time period include the time necessary either to trade machinery for labor at its beginning or bread for machines at its end, so that production time includes whatever time is necessary for exchange as well as production. Let the two exchange times be equal so that neither trading commodities nor shifting factors is preferred on the basis of a net saving in time. Suppose further that all plans involve attaining a fixed and constant level of consumption, and no further saving, within six time periods. Finally, suppose that the supply of labor is constant.

The commodity and factor spaces have 12 dimensions each in this example. The indifference hypersurface in either space will thus appear as a twelve-dimensional bowl. When the individual is in equilibrium, this bowl will be tangent to an eleven-dimensional budget hyperplane at a point. That point may be viewed in ordinary two-space by taking slices of the twelve-dimensional hyperspace, parallel to its two-dimensional faces, at the point of tangency. Twelve-space has $n(n-1)/2$ or 66 two-dimensional faces. Examples of the axes of one of these faces for the commodity hyperspace

would be, for instance, machines at time t=3 and bread at time t=5. Another would be machines at time t=2 and bread at time t=2. Similarly, the corresponding slice of the factor-service indifference hypersurface would have machinery and labor at time t=2 on its two axes.

Figure 10 looks at the six slices which take bread and machines and labor and machinery for the same years and shows six views of a point on the indifference hypersurface for consumption, and six views of the corresponding point on the factor-service indifference hypersurface.

Six lines traverse the bread-machine space. They are slices of the eleven-dimensional budget hypersurface tangent to the indifference hypersurface defined for amounts of consumption and investment each year. Each slice is made at the given point where the budget and indifference hypersurfaces are mutually tangent. The point $d_{(o)}$ is a view of this point, in the slice corresponding to time t=o. In that two-space, we see a slice, I, of the indifference hyperplace, and it is tangent at $d_{(o)}$ to the line $e_{(o)}d_{(o)}$, which is the corresponding slice of the budget hypersurface. From $d_{(o)}$ a broken line makes its way to the terminal $d_{(5)}$, which shows consumption of bread and machines in the 6th and all subsequent years, after the completion of the investment plan the diagram represents. Each break of the path $d_{(o)}d_{(5)}$ shows the availabilities to the country of bread and machinery, the latter for both consumption at the end of the given production period or for investment at the beginning of the next.

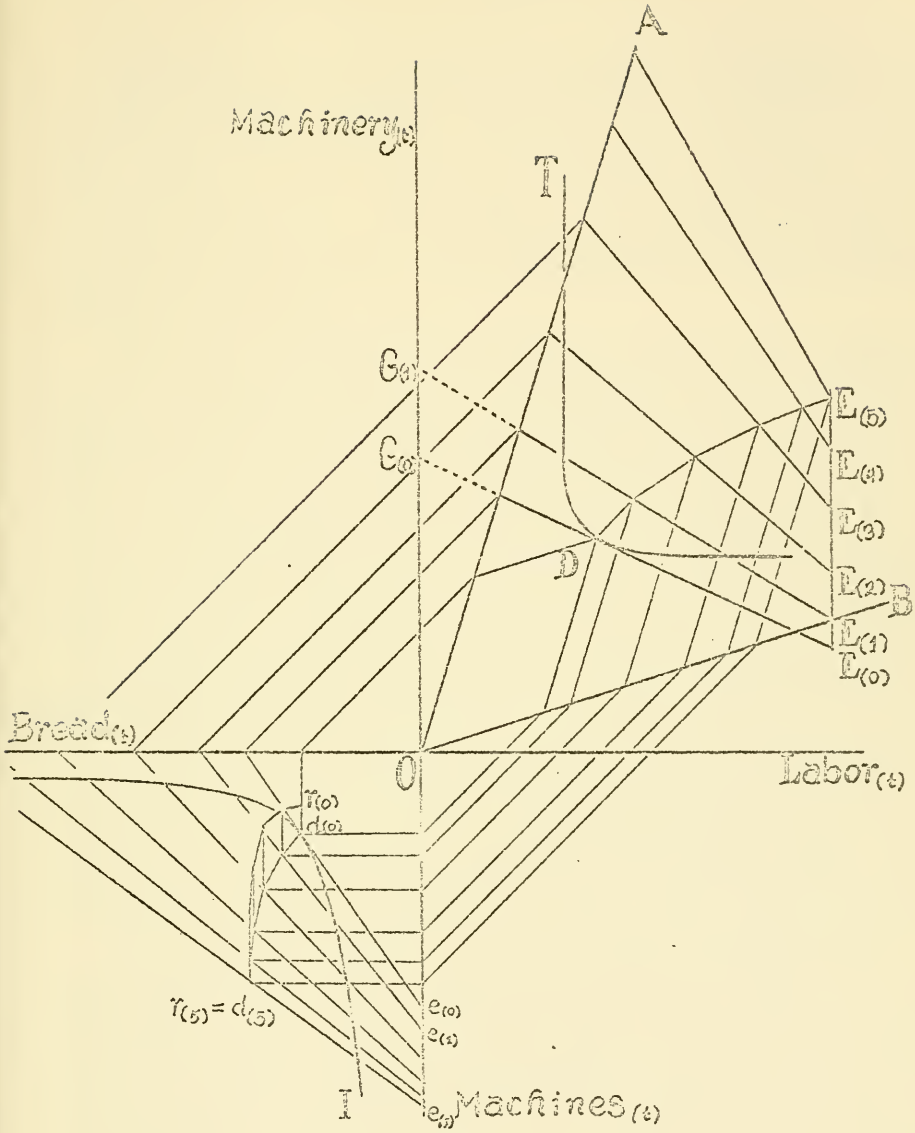
A
Machinery(5)
T
G(5)
G(0)
E(5)
E(4)
E(3)
E(2)
E(1)
E(0)
D
B
Bread(1)
O
Labor(1)
r(0)
d(0)
r(5) = d(5)
e(0)
e(1)
e(5)
I
Machines(1)

FIGURE 10

The path $r_{(o)}r_{(5)}$ shows the time path of consumption in this six-year program. Vertical distances, like $r_{(o)}d_{(o)}$ for the initial year, between the two paths show the amounts allocated to investment in each time period. These amounts are determined according to the production functions for the two commodities and the individual's own time preference.

The first quadrant of the diagram shows how the former enter into the picture. For graphical simplicity, it is assumed that no substitution of labor for machinery takes place within the range that relative prices cover throughout the planning period. The ratio of labor and machinery in the production of bread is given by the vector A, that for machines by vector B. The budget lines from the third quadrant can be mapped into corresponding budget lines in the factor space in the usual way by means of parallel lines, one set of which is shown connecting points along vector A with points along the bread axis. Thus the lines emanating from the points $E_{(o)}$ though $E_{(5)}$ are slices of the eleven-dimensional budget hyperplane showing the amounts of factor services a given outlay can purchase.

Each slice, like I, of the indifference hypersurface in commodity space maps into a corresponding slice, like T, of the factor hyperspace. The point D in the diagram is a view of the tangency between the factor-service indifference hypersurface and the budget hypersurface. The broken path $DE_{(5)}$ connects points showing five other such views. A similar path will characterize each point on

the factor-service hypersurface. Each such point thus will represent a complete "development program," the optimal program, we assume, for the relative prices given by the slopes of the budget hypersurfaces and for the individual's tastes.

The investment part of the program shows the connection between the consumption path, $r_{(o)}r_{(5)}$, and the paths showing total outputs or availabilities and total inputs in each period. The distance $r_{(o)}d_{(o)}$ is equal to the distance $E_{(o)}E_{(1)}$, for instance, and the subsequent vertical distances between the path from $r_{(o)}$ and the path from $d_{(o)}$ correspond, respectively, to vertical distances $E_{(1)}E_{(2)}$, $E_{(2)}E_{(3)}$, etc. It is assumed that the amount of labor employed remains the same throughout the planning period, so that $E_{(o)}E_{(5)}$ is a vertical straight line.

The particular program depicted than proceeds as follows. Initial endowment is at $E_{(o)}$, let us suppose. At the beginning of year 0 the country exchanges the services of labor for those of machines until point D is reached. That year's production is $d_{(o)}$, which point is connected to point D in the usual manner, as shown. A number of machines, represented by $d_{(o)}r_{(o)}$, is defalcated from current production for investment at the outset of period 1. This means that the country then exchanges from the point $E_{(1)}$, along the line emanating from that point, the slope of which indicates the terms of the exchange. In fact, since it already had invested the number of machines corresponding to the ordinate of the point D in the previous year, the second year's transactions consist of

recalling some labor from the other country and adding the new machinery to it directly. It is possible that some machinery will be imported from the other country also in accordance with the improved terms of factoral trade. This depends upon the vertical distance between point D and the point at the next break in the path leading from D to $E_{(5)}$.

From a point like $E_{(1)}$, or the endowment points lying above it which lie on or in the equalization region special to country A, the country could achieve the same effect through commodity trade that it has just been described as doing through the exchange of agents of production. If endowments are at $E_{(1)}$, for example, it produces at $e_{(1)}$ and trades, at the end of the production period, to $d_{(o)}$. In this trade, it produces more machines than it consumes and invests, and buys bread with the surplus. Not all points within the equalization region will be equivalent, of course, from the point of view of joint production of the two commodities so that, as we have seen, the actual locus of indifference betwen the two types of exchange, under protection, will be some line traversing the space between the vectors A and B (refer to Figure 3 and the discussion there).

The rate of interest in each time period can be read from the diagram almost directly. It is equal to p_{oi}/p_i. The maximum number of machines the country could rent with its income at D, say, is the amount $OG_{(o)}$, or $M_{(o)}$. The line $E_{(o)}$ D has been extended to $G_{(o)}$, on the machinery axis. The maximum amount of machines it could buy

with the same income is given by the distance $Oe_{(o)}$, or $m_{(o)}$. Thus income is equal to

$$Y_{(o)} = M_{(o)}p_{o2(o)} = M_{(o)}i_{(o)}p_{2(o)} = m_{(o)}\ p_{2(o)}$$

where machines are taken as the second commodity and machinery as the second factor. This means that

$$i_{(o)} = \frac{m_{(o)}}{M_{(o)}} = \frac{Oe_{(o)}}{OG_{(o)}} \quad ; \quad i_{(1)} = \frac{Oe_{(1)}}{OG_{(1)}} \ ; \text{ etc.}$$

The diagram thus gives the time paths of incomes, interest rates, relative prices, endowments, investment, consumption, trade, migration, and foreign investment as functions of the slopes of the budget hyperplace and the initial endowment point, $E_{(o)}$. It thus represents in itself a complete "model" of trade and accumulation. In this particular model, food has been taken as the capital intensive product, and also the one with income elasticity of demand less than one. Both assumptions fit the real world very closely, and illustrate the crucial importance of factoral trade to a country beginning with a low stock of capital per worker. If the country were restricted, or restricted itself, to the point $E_{(o)}$ and the factor-service indifference hypersurface tangent to that point, a slice of which would be well below the slice T, for example, its food income might well be too low to permit any net saving, although the interest rate, it can be seen, would be very high.

It can also be seen that such a country, at the initial stages, would derive the greatest benefit from producing and exporting a commodity it has little use for in consumption. The positive correlation, which holds strongly in the real world, bewteen labor intensity and income elasticity of demand makes large amount of trade desirable as between rich and poor countries. The ordinary prejudice against trade on both sides, is thus particularly detrimental to poor countries.

The model of Figure 10 generalizes quite easily and directly to many commodities, both for investment and consumption or for either, and to primary factors other than labor and various types of machines, etc. If goods differ in their production periods, it is necessary to add more sectors even if only two final goods do exist. The length of the time period must not exceed the time required to produce the commodity which requires the least time, but it can be less. The other commodities can be only partially completed in that span, of course, but their unfinished stages can be considered as intermediates for the following stages. From the preceding sections of this paper we know how to handle any number of intermediates, even if they are untradable, as half-baked things may often be, or noncompetitive, and so there is no additional conceptual difficulty. The equations for the multi-commodity model are as follows:

(1) $$1 = \Psi_j[f_{j(t)}, a_{j(t)}];$$

(2) $$p_{ok(t)} = \varphi_{kj}[f_{j(t)}, a_{j(t)}] \text{ and}$$

$$p_{i(t)} = \varphi_{ij}[f_{j(t)}, a_{j(t)}];$$

(3) $$[I - A_{(t)}]\, x_{(t)} = r_{(t)} + [f_{(t+1)} - f_{(t)}] = d_{(t)};$$

(4) $$F_{(t)}x_{(t)} \leq f_{(t)} \quad .$$

The vector of final consumption in each of the T time periods is $r_{(t)}$. Using these equations, one can establish the equivalence

$$W[r_{(t)}] = I[d_{(t)}] = T[f_{(t)}] \, .$$

The interest rate in the multi-commodity model is equal to

$$i_{(t)} = \frac{p_{oj(t)}}{p_{j(t)}} \qquad (j = 1, \ldots, n) \quad .$$

It therefore determines, potentially, (n-1) relationships between the prices of commodities usable as capital goods and their rentals when so used in each time period, as soon as the rental and the price of one such commodity are determined. This property enables one to use the interest rate itself as a surrogate for the returns to a metaphysical factor "capital" for certain purposes.

10. How to measure protection

The close and ifficult reasoning of this paper has been necessary only to show a very simple way of dealing with international trade, investment, migration, etc. The factor-service indifference curve can be used like an ordinary indifference curve in the box diagram of elementary theory. Its use makes it legitimate to consider the dimensions of the box fixed as relative factor prices change in both countries. The box can be stretched in the appropriate ways to include the accretion of capital, or of any other factor measuring its dimensions. All commodities are intermediaries between primary factors and enjoyment, so problems of intermediate commodities and of untradable commodities wash out. The only complexity is that the exchange of goods and that of factors will be perfectly equivalent only within a given subspace of the equalization regions of the two countries. We have analyzed this subspace when there are two factors and two commodities. With more commodities and in higher dimensional factor space, these intersections are more complicated, but less restrictive.[12]

This paper has treated only tariffs and direct taxes on the movement of factors. Factor-service indifference curves can be derived for any given set of internal taxes and subsidies, however, and these can be used in the Edgeworth-Bowley boxes. In some cases, taxes and subsidies on domestic production will permit the country to attain a higher consumer indifference curve than tariffs, for consumers

in both countries can trade at a common set of relative prices. Intermediate goods make it necessary to charge duties to their users, but not their final consumers, in order to assure that the country reaches its highest factor-service indifference curves.

Countries also use a variety of quotas and other explicit and implicit restrictions to keep from trading the services of basic factors. The variety of restrictions is an indication that countries do not maximize anything through trade policy, but that variety has made it impossible to assess the damage protection does. The factor-service indifference curve can measure that damage easily and directly.

It is well known that an average duty is worse than none if one wants to know how protectionist is the country applying it. High duties prevent imports and fail to be counted. Similarly, it is the potential trade between countries like India and the United States or Japan and Argentina which really measures the losses of protection. Although the factor-service indifference curve can be used to measure the gains of trade between anytwo countries, its best use is in the extreme cases.

To illustrate its use, therefore, let us apply it to Indo-American trade. The United States is India's best customer, but India's total exports are only about 3 percent of its very low income, so one could say that the two countries effectively do not trade or exchange between them the services of land, labor, and capital either directly or indirectly.[13]

American restrictions take the form of a prohibition on immigration and high tariffs and other restrictive measures applied to labor-intensive exports, particularly textiles. Because other countries sell in the United States many of the things India should sell there, however, it is clear that India's own restrictions are more telling than America's. The decision to channel most investment into import substituting, capital-intensive industries, including agriculture, is one of the most serious. Modern labor-intensive industries working for export, like electronics, ready-made clothing, furniture, hardware, ceramic products, motorcycles, machine tools, watches, toys, sporting equipment, etc. have not been developed or allowed to develop. The emphasis on heavy industry, which by and large means industries producing intermediate products, has starved the industries at the higher stages of manufacture for inputs at reasonable prices and this erodes their capacity to sell on the world market. Maintaining a heavily overvalued exchange rate does the same thing, and in addition prevents the inflow of foreign capital. When devaluation is constantly immanent the foreign investor will tend to "wait one more day" before coming in, even though he may end up wishing he had gone in right away. If he does come in, he must wonder whether he can keep on repatriating his capital through the exchange crisis, for there is an obvious danger that Government will be unable to permit repatriation unless the foreign investment is forthcoming in the first place. Requirements to take on local collaborators and to comply with a host of internal

restrictions, together with the uncertainty of supply of raw materials and spare parts compound the fear of investing in India. Thus there is a vicious circle that effectively stops the inflow of foreign capital.

The cost of not trading factor services can be measured with the aid of Figure 11. The curve T^A is a factor-service indifference curve for the United States, the curve T is India's. It is possible to consider these curves as level curves of the Cobb-Douglas production functions of the two countries. The exponents of these production functions differ in the two countries, however. The share of property is larger, and that of labor smaller, in India than in the United States. Thus the underlying utility function in factor-services cannot be Cobb-Douglas, although its isoquant-map may be considered as a distorted Cobb-Douglas isoquant-map. In the figure, for instance, the curve T^A is shown tangent at point a to line ta. The dotted line At connects all the points where it is supposed that isoquants have the same slope as at t. In a Cobb-Douglas function, all these points would lie on the same ray through the origin, rather than on this bent line. The bent line indicates that food is more important at low levels of income and consumption and that food is relatively capital intensive.[14]

Although curves T and T^A shown in the diagrm are guesses of the actual curves, certain things are known about them. Curve T is tangent to the line ub at u, and ub is parallel to ta. Points a

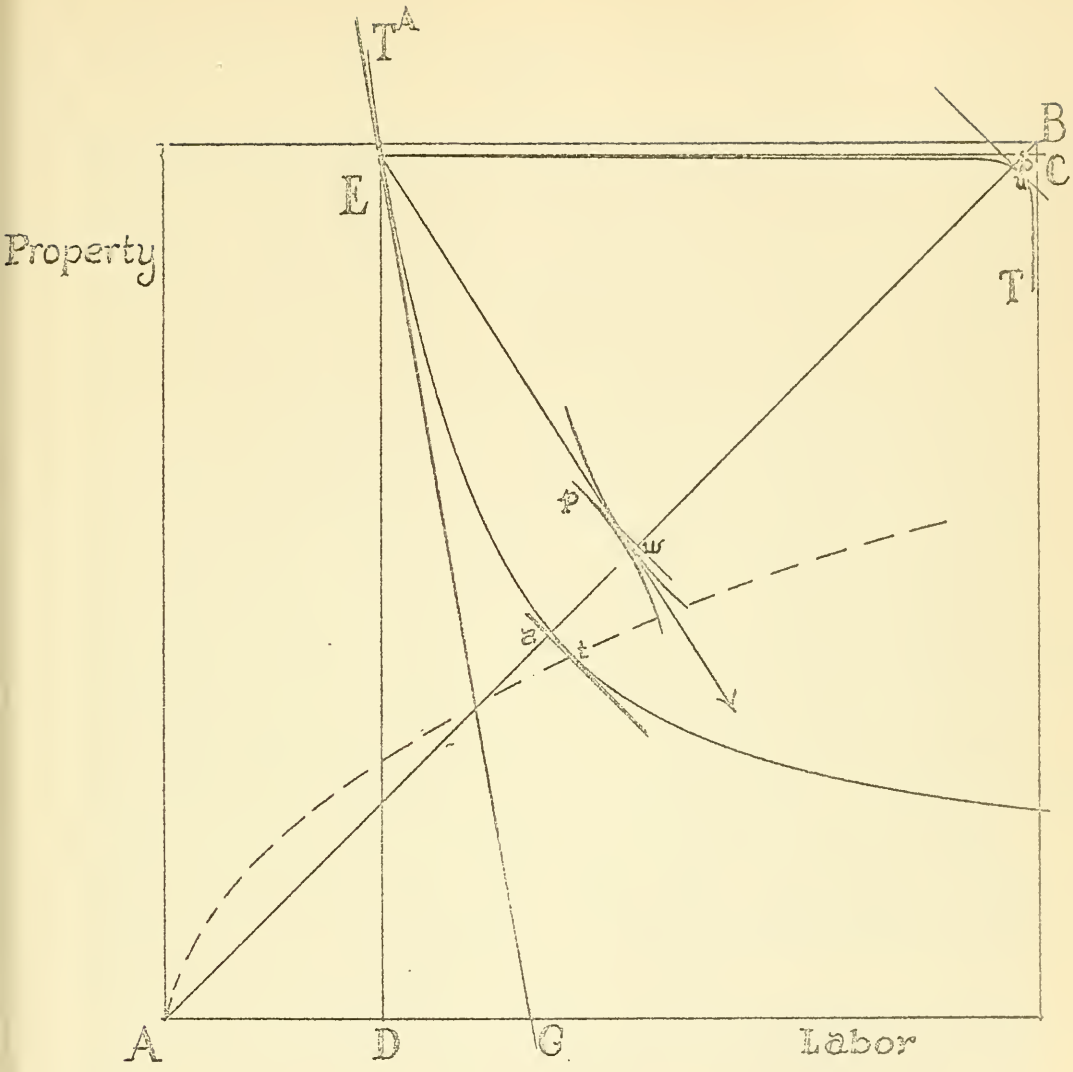

FIGURE 11

and b lie on the line segment AB which connects the two origins. The distances Aa and Bb therefore measure the relative incomes of the two countries. Per capita income in India is generally considered to be about one thirtieth to one fiftieth of that in the United States, and so Indian national income is, say, about one sixteenth of American. Thus the distance Bb is shown as one sixteenth of the distance Aa.

The two factor-service indifference curves must be placed in a reasonable way and given a reasonable shape that preserves the relative positions of a and b on AB. Two other things are known about them that help to tie them down more firmly. We know the share of labor and property, which includes land and all types of capital under the assumption that domestic prices and incomes more or less reflect true scarcities, in the two countries, although official estimates must be adjusted somewhat for the present exercise. We also know the amounts of labor in the two countries and that the United States employes about one quarter of the joint total. The American supply is shown as AD, which distance is one quarter of the length of the bottom border of the Edgeworth-Bowley box AB.

If we had capital estimates for the two countries, the point E, which indicates both the endowment and the consumption of the two factors for each country, could be placed directly. But it must be placed fairly closely by knowing the slopes of the curves at that point. These can be taken from knowledge of factor shares.

A factor's share in national income can be represented easily in the basic diagram of this paper. It is important in its own right

as a measure of protection. In Figure 1 the point N, lying on the line 1-2 extended to intersect the vertical axis, shows how much machinery the national income could rent if it were spent entirely on that factor. The point M is the machinery-coordinate of the point D, which point indicates the consumption of the two types of factor services, and M thus shows how much machinery is actually rented. The ratio OM/ON therefore gives the share of machinery in national income. The same ratio in Figure 2 shows the effects of the protection of labor which, by making the tangent at D more steep, has decreased machinery's share. Machinery's income may have increased, however, because of the improvement in the factoral terms of trade. In Figure 11, labor's share is AD/AG, which ratio is taken as known. This immediately gives the slope of the line EG and therefore of the tangent to the American factor-service indifference curve at E. Similarly, the slope of the curve T at E is given by that of the line tangent to T at that point. That line will cross the property-axis of India at a point beyond C from B, and BC is India's supply of property. Thus that intersection, which is not shown, in conjunction with point B, measures the relative share of property in Indian national income, just as G measures the share of labor in American national income.

What are these shares? Generally labor is said to receive between 65 and 70 percent of American income, but clearly part of this income is the return on the considerable capital invested in

labor skills. That capital is considered to be about 20 or 25 percent of the total capital, and so labor's proper share has been reduced to 60 percent in the diagram.

A somewhat different adjustment must be made for Indian labor. Because so much of it is unemployed, its economic value should be taken as zero, in which case the curve T would pass through E with zero slope. On the other hand, labor commands a wage in India and its share in national income is taken to be about 40 percent in most reports. Why does labor command a wage when it is in excess supply? There are many possible and well known answers, of course, but it seems to me that the most important is that an Indian worker needs to eat considerably more to work than simply to live, and that most, if not all, of his wage is, from a strictly economic point of view, simply a disbursement for an intermediate commodity, food. If the worker possesses any skill it will pay his employer to pay him a livable wage rather than to incur the constant expense of training new workers who will live only a short time. Landless laborers in the country, whose average wage is only some Rs 8 per month, compared to the Rs 4 per day which factory workers earn, clearly earn less than a livable wage, and their life expectancy is very short. They can easily be replaced by people already capable of doing the work and their short life wastes little investment. So, in using the wage rate as the economic rent of the factor labor, the cost of subsistance should be subtracted. Doing this leaves little in India for labor, but labor's share has been taken as about one third in Figure 11 for

conservatism's sake. Obviously any such adjustment makes negligable difference. The essentials of Figure 11 follow simply from the enormous difference in income as between the United States and India and the share of labor in the United States.

The arrow through E shows what the terms of trade would probably be under the equivalent of free factoral trade between India and the United States. This arrow goes through the point where factor-service indifference curves of the two countries are tangent. From the point p on India's curve, line pw, parallel to ta and bu, has been drawn to point w on AB. Point w measures the income which India would have under the equivalent of free factoral trade if the general configuration of the curves were as shown.

Something can be said about this general configuration from the knowledge that food would become of lesser relative importance in the incomes of both countries if their incomes increased. Because leisure and luxury goods are both relatively labor-intensive, the joint demand for labor in the world would grow with the increase in world income which the figure shows to be possible, no matter how that increase were distributed. Thus the wage of labor would not fall by nearly so much in the United States as it would rise in India, and this means that India's absolute gain would be larger than America's.

India's relative gain, it can be seen, cannot fail to be enormous. The point w represents an increase of some 16.5 times present Indian income, while a similar point for the United States would represent

a gain of only about 12 percent in American national income. Any reasonable rendering of Figure 11 would lead to the same conclusion that present international economic policies practically eliminate Indian income.

1. Many important questions in international trade, especially trade in second-hand machinery and the problem of externalities (learning-by-doing, etc.) can be handled best by allowing for joint production. I am working on this.

2. See W. P. Travis, The Theory of Trade and Protection, Cambridge, 1964, pp. 37-38.

3. The curve T-T exists even if the isoquants cross each other more than once. At one point on I-I, the commodity price plane will determine a factor price plane tangent to both isoquants in the same point. That point will divide the indifference curve into two segments. As d moves outward along one of them, it will map into two points, one on either side of the line connecting the critical point, where the two isoquants are tangent, with the origin. Each point will trace out a branch of the T-curve. Along the other segment of the indifference curve, d will map into points successively farther and farther from the origin along the line through the critical point. The entire T-curve will have, thus, the appearance of an inverted mushroom. Factor prices will be the same all along its stem, and equal to the slope of the two isoquants at their tangency.

4. If country A sets her tariff according to p^B, which corresponds to actual practice, the market will not distinguish between p* and p^B because each consumer will pay a tiny part of the total customs duty and will not perceive the reduction that his foreign purchase makes possible in, say his income tax.

5. Segment Jf is equal to $k(1^B)$ in the bottom diagram. Segment ak there is parallel to $1^A(1^B)$ and to the line connecting the intersections of the extensions of vectors 1^A and (1^B) with the isoquant Q_1.

6. An intriguing unimportant case arises when factor-intensities switch over. Then it is possible that the two country's equalization regions fail to intersect at all, so that factor migration, separately balanced of course, has to take care of all factoral trade.

7. See W. P. Travis, The Theory of Trade and Protection, op. cit., pp. 131 n. 6, 135, and 138-139.

8. It is misleading, moreover, to say, with L. W. McKensie and others, that they shift outward the world production possibilities curve. They will not have this effect if factor endowments lie in some sub-equalization region, drawn up on the basis of direct-plus-indirect coefficients, like region $OA(l_b + l_a)B$ of Figure 6. When factor endowments lie outside of this equalization region, but inside the one defined by means of the direct input coefficients, movement of intermediates permits attaining the world transformation curve without movement of primary factors among countries.

9. Corden argues that untradable inputs should be treated as primary factors. He reports that Basevi and Belassa treat them as if they were imported, but subject to no duty. Both positions reveal fundamental misconceptions. Corden is unclear about the definition of primary factors. Basevi and Belassa, in their

measurements, should have assigned a tariff to the untradable inputs reflecting their actual cost differences in various countries, or else, melding in their input coefficients with those of the user industries in the manner outlined here, constructed composite industries to eliminate the industries producing controlable intermediate products. See Corden, W. M., "The Structure of a Tariff System and the Effective Protective Rate," The Journal of Political Economy, Vol. LXXIV, No. 3, (June 1966) pp. 226-228.

10. Irving B. Kravis, "Availability and Other Influences on the Commodity Composition of Trade," Journal of Political Economy, 64:143 - 155 (April 1956).

11. W. P. Travis, The Theory of Trade and Protection, op. cit., pp. 150-151, 163-169.

12. In three dimensions, for instance, the intersection is a plane. If there are three commodities and only two factors, the intersection will be the projection of this plane onto two-space, and will thus be an area, rather than a line segment, in that space.

13. A country can import land directly by exporting capital and labor to the other country and combining the three factors there in the same proportions, for any given occupation, as those in which they would be combined at home. This would give rise, in effect, to an enclave in the other country.

14. This is a fact of obvious and great importance in trade and development, but it is relatively little documented in economic literature. It appears in Leontief's estimates of American

input coefficients. See also Paul Bairoch, Revolution industrielle et sous developpment, Paris (SEDEIS) 1963, pp. 117-118 for an assemblage of material giving historical data on the relative capital intensity of agriculture and manufacturing for various countries.

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